4.3 Connecting f ' and f '' with the graph of f

first derivative test for local extrema of continuous functions

If $f' > 0$, f increases
If $f' < 0$, f decreases

f': $+ \; 0 \; -$ f has a max
or $+ \; * \; -$

f': $- \; 0 \; +$ f has a min
or $- \; * \; +$

f': $+ \; 0 \; +$ no extrema
$- \; 0 \; -$

left endpoint: $f' > 0$, f has min
$f' < 0$, f has max

right endpoint: $f' > 0$, f has a max
$f' < 0$, f has a min

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find the local extrema:

y=(x^2 - 3) e^x

$y' = (x^2-3)e^x + e^x \cdot 2x = 0$ $*$ none

$e^x(x^2-3+2x) = 0$

$(x+3)(x-1) = 0$

$x = -3, \; x = 1$

f': $+ \; 0 \; - \; 0 \; +$ (at -3 and 1)

max at $x=-3$
because f ' changes from pos to neg

min at $x=1$
because f ' changes from neg to pos

max at $x=-3$
max is $y = 6 \cdot e^{-3}$
min at $x=1$
min is $y = -2e$

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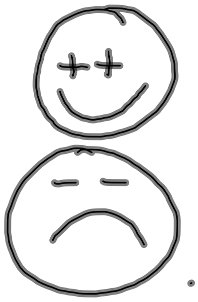
concavity test

$f'' > 0$ then f is concave up

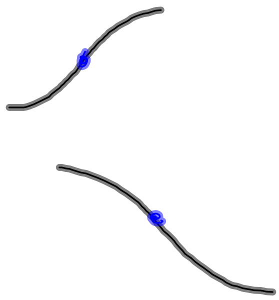
$f'' < 0$ then f is concave down



f'' changes sign : f has an inflection point

$+ \ 0 \ -$

$- \ 0 \ +$



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Find all points of inflection for the graph of $y = e^{-x^2}$

$y' = (-2x)e^{-x^2}$

solve for x

$y'' = (-2x)(-2x)e^{-x^2} + e^{-x^2}(-2) = 0$

$y'' = e^{-x^2}(4x^2 - 2) = 0$

$4x^2 - 2 = 0$

$4x^2 = 2$

$x^2 = \frac{1}{2}$

$x = \pm\sqrt{\frac{1}{2}}$

y''	$+$	0	$-$	0	$+$
		$-\sqrt{\frac{1}{2}}$		$\sqrt{\frac{1}{2}}$	

Inflection points at $x = -\sqrt{\frac{1}{2}}$, $y = e^{-\frac{1}{2}}$

$x = \sqrt{\frac{1}{2}}$, $y = e^{-\frac{1}{2}}$

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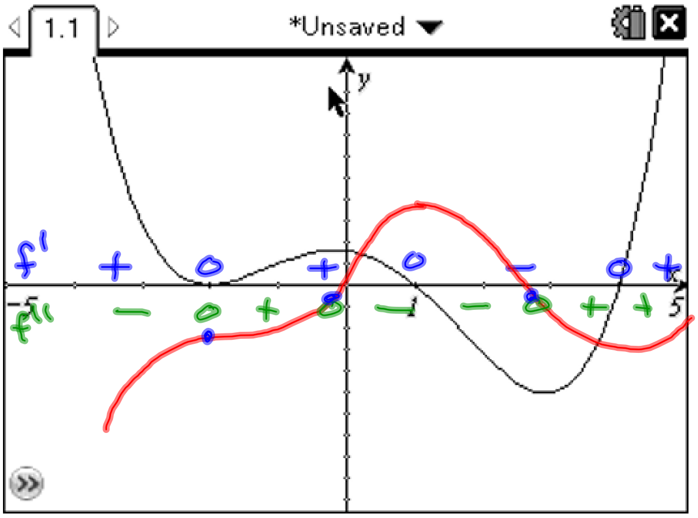
This is the graph of f '. Sketch a possible graph of f



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second derivative test for local extrema

if $f'=0$ and $f''>0$ f has a min

if $f'=0$ and $f''<0$ f has a max

Find the local extrema using the second derivative test

y=x^3 -12x - 5

$y' = 3x^2 - 12 = 0$

$x = \pm 2$

$y'' = 6x$

$y''(-2) = -12 < 0$

y has a max at $x=-2$

$y''(2) = 12 > 0$

y has a min at $x=2$

max is $y(-2) = 11$

min is $y(2) = -21$

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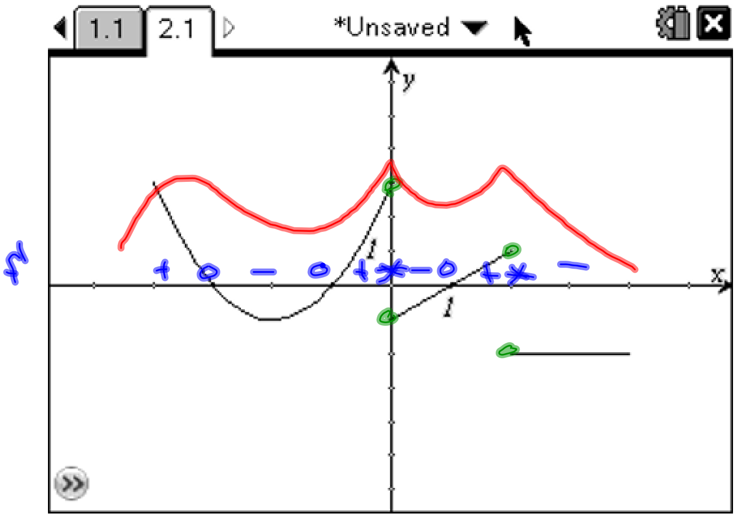
Given the graph of f ' sketch a possible graph of f



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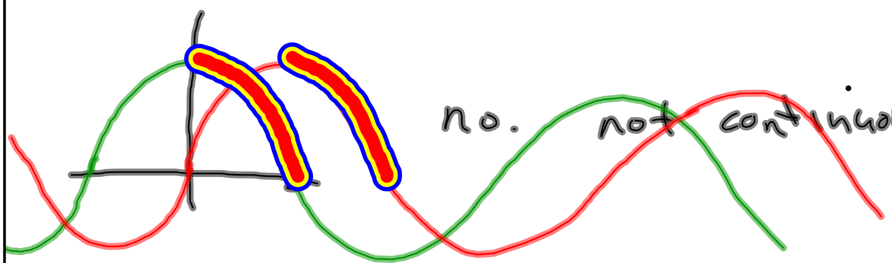
7. $f(x) = \begin{cases} \cos x, & 0 \le x < \frac{\pi}{2} \\ \sin x, & \frac{\pi}{2} \le x < \pi \end{cases}$



no. not continuous

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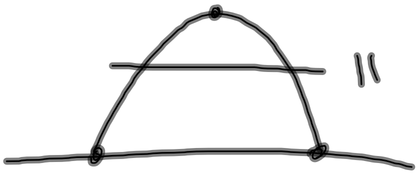
14. $\frac{26.2-0}{2.2-0} = 11.9 =$ ave vel

at least once, runner went 11.9



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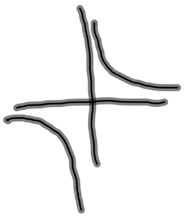
17. local extrema, inc, dec

none — no where — $(-\infty,0) \cup (0,\infty)$

$h(x) = \frac{2}{x}$



$h'(x) = \frac{-2}{x^2} = 0$ never

$- - * - -$ at 0

$h'(x) \neq 0$

$h'(x) = *$ at $x=0$ but $h(0) = *$

no critical points

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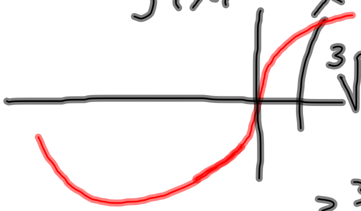
min at $x = -2$
min is $\sqrt[3]{-2} \cdot 6$

$[-2, \infty)$ $(-\infty, -2]$

24. extrema, inc, dec

$$g(x) = x^{\frac{1}{3}}(x+8)$$

$$g'(x) = x^{\frac{1}{3}} \cdot 1 + (x+8)\frac{1}{3}x^{-\frac{2}{3}} = 0$$



$$\left(\sqrt[3]{x} + \frac{x+8}{3\sqrt[3]{x^2}} = 0\right) 3\sqrt[3]{x^2}$$

$$3\sqrt[3]{x^3} + x + 8 = 0$$

$$4x + 8 = 0$$

$$x = -2$$

f' − 0 + ✱ +

−2 −1 0 1

other critical pts? $x = 0$

Oct 20-9:30 AM